The optimal realizations of a PID vibration suppression and disturbance rejection controller has been investigated by considering implementations when the blocks are grouped into the A, B, C and D matrices of the state space description even though the realizations are optimal with respect to equal rounding on all the state space parameters.

Controller Parameter Representations

Fixed Point

Each parameter $p$ with fixed point representation has the form $p = p_{\min} + p_{\max} \cdot 2^{-\ell}$, where $p_{\min}$ is the integer part and $p_{\max} < 1$ is the fraction. The implementation is usually based on the assumption that the location of the binary point is fixed, and the representation of the numbers is commonly by two’s complement.

The maximum rounding error on a parameter, $\Delta$, is given by $|\Delta| < 2^{\ell-1} - 1$ where $2^{\ell}$ is the fraction wordlength.

Block Floating Point

A matrix $P$ in BFLP representation has the form

$$ P = \tilde{P} + \Delta P $$

where $\tilde{P}$ is called the exponent and $\Delta$ the block mantissa of $P$.

Two possible block floating point implementations are considered:

i) there is a single block exponent, the block being the matrix $P$ of the controller is a block, this implementation is referred to here as "single BLFP"

ii) each matrix $A_k, B_k, C_k, D_k$ of the controller is a block, this implementation is referred to here as "ABCD BLFP"

Finite Precision Stability Measures

When $(A_k, B_k, C_k, D_k)$ is implemented with a digital computing device, $X$ is perturbed to $X + \Delta X$ due to rounding effects. Define

$$ \mu(\Delta X) \triangleq \max_{i \in \{1, \ldots, m+n\}} \frac{1}{|\lambda_i|} \left| \sum_{i=1}^{m+n} \lambda_i \right| $$

where $N$ is the number of controller parameters and define a finite precision stability robustness measure as

$$ u_p(X) \triangleq \min_{\Delta X} \mu(\Delta X) : X + \Delta X \text{ is unstable} $$

The round off error that the closed loop system can bear increases with increasing $u_p(X)$. Several tractable measures that are lower bounds of $u_p(X)$ have been proposed:

Stability measure $\mu_1$

$$ \mu_1(X) \triangleq \min_{\Delta X} \left| \sum_{i=1}^{m+n} \lambda_i \right| $$

where $\lambda_i$ is any eigenvalue.

Stability measure $\mu_2$

$$ \mu_2(X) \triangleq \min_{\Delta X} \sqrt{\sum_{i=1}^{m+n} |\lambda_i|^2} $$

Stability measure $\mu_3$

Measure based on structured $\ell_1/\infty$ small gain stability theory.

Industrial Example: A Steel Rolling Mill PID Controller

A PID vibration suppression and disturbance rejection controller has been designed as

$$ K(z) = 0.000269 \cdot \frac{0.00435\cdot s + 1}{s} $$

The optimal realizations of $K(z)$ with respect to the stability measures have been calculated:

<table>
<thead>
<tr>
<th>Controller Parameter</th>
<th>Fixed-point</th>
<th>BLFP</th>
<th>BFLP</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_1$</td>
<td>1.745</td>
<td>1.704</td>
<td>1.745</td>
</tr>
<tr>
<td>$T_2$</td>
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<td>8.919</td>
<td>8.926</td>
</tr>
<tr>
<td>$T_3$</td>
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<td>4.356</td>
<td>4.377</td>
</tr>
<tr>
<td>$T_4$</td>
<td>7.706</td>
<td>7.479</td>
<td>7.706</td>
</tr>
</tbody>
</table>

Comparative stability measures for different optimal realizations

<table>
<thead>
<tr>
<th>Controller Parameter</th>
<th>Fixed-point</th>
<th>BLFP</th>
<th>BFLP</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_1$</td>
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<td>5.844</td>
<td>5.844</td>
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<tr>
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<tr>
<td>$T_3$</td>
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<td>4.218</td>
<td>4.227</td>
</tr>
<tr>
<td>$T_4$</td>
<td>6.786</td>
<td>7.227</td>
<td>6.786</td>
</tr>
</tbody>
</table>

Minimum stabilizing average wordlengths per parameter for different realizations

The performance sensitivity is investigated by considering implementations using a maximum total wordlength of 8 bits. The frequency responses of the original infinite precision closed loop system $R(z) = G(z)K(z) + G(z)\bar{K}(z)$ and the errors of the finite precision optimal realizations are shown.

Frequency response for realization $T_2$ for fixed point $(-)$, single block floating point $(\cdots)$ and ABCD block floating point $(\cdots)$ implementations

Frequency response for realization $T_3$ for fixed point $(-)$, single block floating point $(\cdots)$ and ABCD block floating point $(\cdots)$ implementations

Stability Analysis of Block Floating Point Digital Controllers

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